Problem:

Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?

Solution:

We shall let *A* be the event that the first fuse is defective and *B* the event that the second fuse is defective; then we interpret *A ∩ B* as the event that *A* occurs and then *B* occurs after *A* has occurred. The probability of first removing a defective fuse is 1/4; then the probability of removing a second defective fuse from the remaining 4 is 4/19. Hence,



Note: If the first fuse is replaced and the fuses thoroughly rearranged before the second is removed, then the probability of a defective fuse on the second selection is still 1/4; that is, *P*(*B|A*) = *P*(*B*) and the events *A* and *B* are independent. When this is true, we can substitute *P*(*B*) for *P*(*B|A*) in the multiplicative rule.

Problem:

One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?

Solution:

Let *B*1, *B*2, and *W*1 represent, respectively, the drawing of a black ball from bag 1, a black ball from bag 2, and a white ball from bag 1. We are interested in the union of the mutually exclusive events *B*1 *∩ B*2 and *W*1 *∩ B*2.

The various possibilities and their probabilities are illustrated in the following Figure:





Problem:

A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Solution:

Let *A* and *B* represent the respective events that the fire engine and the ambulance are available.

Then *P*(*A ∩ B*) = *P*(*A*)*P*(*B*) = (0*.*98)(0*.*92) = 0*.*9016*.*

Problem:

Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Find the probability that the event *A*1 *∩ A*2 *∩ A*3 occurs, where *A*1 is the event that the first card is a red ace, *A*2 is the event that the second card is a 10 or a jack, and *A*3 is the event that the third card is greater than 3 but less than 7.

Solution:

First we define the events

*A*1: the first card is a red ace,

*A*2: the second card is a 10 or a jack,

*A*3: the third card is greater than 3 but less than 7.

Now



And hence, by using Multiplication Law,



Problem:

In a certain assembly plant, three machines, *B*1, *B*2, and *B*3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

Solution:

Consider the following events:

*A*: the product is defective,

*B*1: the product is made by machine *B*1,

*B*2: the product is made by machine *B*2,

*B*3: the product is made by machine *B*3.

Applying the rule of elimination,

we can write *P*(*A*) = *P*(*B*1)*P*(*A|B*1) + *P*(*B*2)*P*(*A|B*2) + *P*(*B*3)*P*(*A|B*3)*.*

Referring to the following tree diagram, we find that the three branches give the probabilities



*P*(*B*1)*P*(*A|B*1) = (0*.*3)(0*.*02) = 0*.*006*,*

*P*(*B*2)*P*(*A|B*2) = (0*.*45)(0*.*03) = 0*.*0135*,*

*P*(*B*3)*P*(*A|B*3) = (0*.*25)(0*.*02) = 0*.*005*,*

and hence

*P*(*A*) = 0*.*006 + 0*.*0135 + 0*.*005 = 0*.*0245*.*